Likelihood ratio analysis of the theoretically interesting effect in the ego depletion study

For testing the relative fit of two models that make contrasting predictions, one approach is to compare each model's fit to a model based on the observations. The likelihood ratio (λ) comparing the fit of the two models of interest will favour whichever model competes best with a model based on the observations. In the null vs. theoretically interesting effect (TIE) analysis, the likelihood ratio favouring the null will be:

$$\lambda(\textit{null vs. tie}) = \frac{\lambda\ (obs\ vs.\ tie)}{\lambda\ (obs\ vs.\ null)}$$

By inverting the equation, we get the likelihood ratio in favour of the tie vs. the null:

$$\lambda(\textit{tie vs. null}) = \frac{1}{\lambda\ (null\ vs.\ tie)} = \frac{\lambda\ (obs\ vs.\ null)}{\lambda\ (obs\ vs.\ tie)}$$

When the sum of squares values from an ANOVA are available, one can use the overprediction method as described in Glover & Dixon (2004, pp. 800-801). In the present case, we do not have the sum of squares values to hand. However, we do have the effect size (.007 or .7%), the *F*-score (4.84), and the sample size (n = 654), which is enough to do the TIE procedure. To start with, we find the value of *F* that corresponds to the deviation of the observations from the theoretically interesting effect, *F(obs-tie)*, computed as:

$$\textit{F(obs vs. tie)} = \left[\sqrt{F(obs)} - \sqrt{F(obs)}\ \left(\frac{TIE}{obs}\right)\right]^2$$

where *F(obs)* is the *F*-score obtained from the ANOVA, *TIE* is the size of the theoretically interesting effect, and *obs* is the size of the observed effect. For a TIE of 2.0% and an observed effect of 0.7%:

$$\textit{F(obs vs. tie)} = \left[2.2 - 2.2\ \left(\frac{2.0\%}{0.7\%}\right)\right]^2$$

$$= 16.69$$

We now have an *F*-score for the observations vs. the null, *F(obs vs. null)* = 4.84, and the *F*-score for the observations vs. the TIE, *F(obs vs. tie)* = 16.69. So, as well as having an *F* that indexes the degree to which the observations deviate from the null, we also have an *F* that indexes the degree to which the observations deviate from the TIE. We then use these *F's* to compute the likelihood ratio for the null vs. TIE models.

The likelihood ratio for the model based on the *observations vs. the TIE* model is:

$$\lambda(obs\ vs.\ tie) = \left(1 + \frac{F(obs\ vs.\ tie)}{df}\right)^{\frac{df+1}{2}}$$

And for the *observations vs. the null* model is:

$$\lambda(obs\ vs.\ null) = \left(1 + \frac{F(obs\ vs.\ null)}{df}\right)^{\frac{df+1}{2}}$$

Algebraically then, the likelihood ratio of the *null vs. the TIE* model is:

$$\lambda(null\ vs.\ tie) = \left(\frac{\left[1+\frac{F(obs\ vs.\ tie)}{df}\right]}{\left[1+\frac{F\ (obs\ vs.\ null)}{df}\right]}\right)^{\frac{df+1}{2}}$$

Inserting the relevant values in this case gives us:

$$\lambda(null\ vs.\ tie) = \left(\frac{\left[1+\frac{16.69}{653}\right]}{\left[1+\frac{4.84}{653}\right]}\right)^{327}$$

$$= 343.1$$

From this we find that the data are 343.1 times as likely given there is no effect than given an effect that is large enough to be theoretically interesting.